

MA 109 Standards

Spring 2023

Standard 1: Functions

Preliminaries

By the end of this lesson, students should be able to

- Plot points on the coordinate plane
- Identify parts of the plane (quadrants, axes, origin)
- Find x- and y-intercepts given either a graph or an equation

Function Notation

By the end of this lesson, students should be able to

- Evaluate a function at a number given the function as an equation, graph, or table
- Solve for when a function is equal to a given number when given the function as an equation, graph, or table
- Evaluate a function given as an equation at an algebraic expression
- Evaluate an algebraic expression involving one or more functions
- Evaluate a piecewise function at a number given that function as a graph or equation

Composition of Functions

By the end of this lesson, students should be able to

- Evaluate a composition of functions at a number, when the two functions are given as any combination of equations, graphs, or tables
- Simplify a composition of functions when those functions are given as equations

Inverses

By the end of this lesson, students should be able to

- Evaluate the inverse of a function at a number, when given an invertible function as an equation, graph, or table
- Determine the equation for the inverse of an invertible function given as an equation
- Determine the domain and range of the inverse of an invertible function, when given that function as a graph

Standard 2: Properties and Operations on Functions

Domain and Range

By the end of this lesson, students should be able to

- Use interval notation
- Determine the domain and range of a function given as a graph and write it using interval notation
- Determine the domain algebraically of polynomials, rational functions, and radical functions and write it using interval notation

Transformations of Functions

The transformations we will consider are vertical and horizontal stretch/compression, vertical and horizontal shifts, and vertical and horizontal flips.

By the end of this lesson, students should be able to

- Given an expression of a transformed function, a graph of a transformed function, or a description of a transformed function, get to any of the other two.

Standard 3: Linear Functions and Applications

Linear Functions

By the end of this lesson, students should be able to

- Determine the slope of a line when given the graph of the line, the equation of the line, or two points on the line
- Write the equation of a line in slope-intercept form when given the slope and the y-intercept, the slope and a point, two points, or the graph of the line
- Write the equation of a vertical or horizontal line

Systems of Linear Equations

By the end of this lesson, students should be able to

- Given a point, determine if it is a solution to a system of two linear equations in two variables
- Determine how many solutions a system of two linear equations in two variables has (where it could have 0, 1, or infinitely many)
- Given a system of two linear equations in two variables that has a unique solution, determine that solution

Linear Models

By the end of this lesson, students should be able to

- Understand slope and y-intercept in the context of real-world problems using linear models in order to write line equations to model linear functions.
- Solve linear applications using Distance = Rate x Time formula.
- Solve mixture problems using a linear model or linear system.
- Model money problems involving sales tax and price reductions.

Average Rate of Change

By the end of this lesson, students should be able to

- Find the average rate of change between two numbers of a function given as an equation, graph, or table
- Find the average rate of change on $[a, a+h]$ or $[\text{number}, \text{number} + h]$ when given a linear or quadratic function as an equation.

Standard 4: Polynomial and Rational Functions

Quadratic Functions

By the end of this lesson, students should be able to

- Solve quadratic equations
- Given x-intercepts and sufficient other information, write the equation of a quadratic function

Polynomial Functions

By the end of this lesson, students should be able to

- Identify the leading term, leading coefficient, and degree of a polynomial function given as an equation
- Determine the end behavior of a polynomial as either a sketch or written with arrows, when given either the equation of the function or sufficient information about its leading term
- Determine whether the leading coefficient of a polynomial is positive or negative and whether the degree is even or odd, when given the graph of a polynomial function
- Determine the factors, roots/zeros/x-intercepts, and their multiplicities when given the equation of a polynomial function in factored form
- Identify the zeros/roots/x-intercepts and whether the multiplicity of each is even or odd, when given the graph of a polynomial function
- Given the factors of a polynomial, determine zeros/roots/x-intercepts, and vice versa
- Write the equation of a polynomial function when given its graph and sufficient extra information
- Write the equation of a polynomial function when given the zeros, their multiplicities, and another point, or other sufficient information

Rational Functions

By the end of this lesson, students should be able to

- Determine the vertical asymptotes when given the formula for a rational function
- Determine the end behavior and whether or not a given rational function has a horizontal asymptote, when given the formula
- Determine vertical and horizontal asymptotes when given a graph of a rational function
- Determine the domain and range of an invertible rational function and its inverse

Standard 5: Exponential and Log Functions

Exponential Functions

By the end of this lesson, students should be able to

- Identify the initial value, and growth/decay rate when given the formula for an exponential function
- Write the equation for an exponential function when given the initial value/y-intercept and either another point or the growth/decay rate.
- Determine the end behavior of a given exponential function which may have transformations
- Determine the initial value and growth/decay rate when given the graph of an exponential function
- Write and evaluate exponential functions when given appropriate information as a word problem

Logarithms

By the end of this lesson, students should be able to

- Write the inverse of a logarithmic or exponential function (without transformations)
- Determine the domain and vertical asymptote of a logarithmic function when given the equation
- Evaluate a composition of a logarithm and an exponential function of the same base

Standard 6: Exponential and Log Applications

Logarithm Properties/Rules

By the end of this lesson, students should be able to

- Expand a single logarithm into a linear combination of logarithms
- Combine a linear combination of logarithms of the same base into one logarithm
- Evaluate/simplify compositions of logs and exponential functions

Logarithmic and Exponential Equations

By the end of this lesson, students should be able to

- Solve logarithmic or exponential equations by conversion to the other.
- Solve logarithmic and exponential equations using One-to-One properties.
- Solve exponential equations with non-matching bases.
- Solve exponential growth models using the techniques above.
- Find the inverse of a function involving exponentials or logs.

Exponential Models

By the end of this lesson, students should be able to

- Determine the doubling time/half-life when given the growth/decay rate of an exponential function
- Determine the growth/decay rate when given the doubling time/half-life of an exponential function
- Determine when some fraction or the original is obtained when given either the growth/decay rate